Algorithm 3 SHGO homology group growth algorithm

1: **procedure** INITIALISATION
2: **Input** an objective function f, constraint functions $\mathbf{g}$ and variable bounds and $[\mathbf{l}, \mathbf{u}]^n$.
3: **Input** N initial sampling points.
4: Define a sampling sequence that generates a set $\mathcal{X}$ of sampling points in the unit hypercube space $[\mathbf{0}, \mathbf{1}]^n$
5: Define the empty set $\mathcal{M}^E = \emptyset$ of vertices evaluated by a local minimisation.
6: **end procedure**
7: **while** $\mathbf{TERM}(\mathbf{H}_1(\mathcal{H}), \min\{\mathcal{F}\})$ is False **do**
8: **procedure** SAMPLING
9: $\mathcal{P} = \emptyset$
10: **while** $|\mathcal{P}| < N$ **do**
11: Generate $N - |\mathcal{P}|$ sequential sampling points $\mathcal{X} \subset \mathbb{R}^n$
12: Stretch $\mathcal{X}$ over the lower and upper bounds $[\mathbf{l}, \mathbf{u}]^n$
13: $\mathcal{P} = \{\mathcal{X}_i \mid \mathbf{g}(\mathcal{X}_i) \geq 0, \forall \mathcal{X}_i \in \mathcal{X}\} \cup \mathcal{P}$ ▷ (Find $\mathcal{P}$ in the feasible subset Ω by discarding any points mapped outside the linear constraints g and adding to the current set of $\mathcal{P}$.)
14: Set $\mathcal{X} = \emptyset$
15: **end while**
16: Find $\mathcal{F}$ from the objective function $f : \mathcal{P} \to \mathcal{F}$ for any new points in $\mathcal{P}$
17: **end procedure**
18: **procedure** CONSTRUCT/APPEND DIRECTED COMPLEX $\mathcal{H}$
19: Calculate $\mathcal{H}$ from $h : \mathcal{P} \to \mathcal{H}$ ▷ (If $\mathcal{H}$ was already constructed new points in $\mathcal{P}$ are incorporated into the triangulation.)
20: Calculate $\mathbf{H}_1(\mathcal{H})$
21: **end procedure**
22: **procedure** CONSTRUCT $\mathcal{M}$
23: Find $\mathcal{M}$ from Definition 20.
24: **end procedure**
25: **procedure** LOCAL MINIMISATION
26: Calculate the approximate local minima of f using a local minimisation routine with the elements of $\mathcal{M} \setminus \mathcal{M}^E$ as starting points. ▷ Process the most promising points first.
27: $\mathcal{M}^E = \mathcal{M}^E \cap \mathcal{M}$ ▷ This excludes the evaluation any element $v_i \in \mathcal{M}$ that is known to be the only point that in the domain $\partial \mathrm{st}(v_j)$ where v_j is known to any point already used as a starting point in Step 27. If any new $v_i \in \mathcal{M}$ not in $\mathcal{M}^E$ is known to be the only point $\partial \mathrm{st}(v_j)$ it can also be excluded.
28: Add the function outputs of the local minimisation routine to $\mathcal{F}$
29: **end procedure**
30: Find new value of $\mathbf{TERM}(\mathbf{H}_1)(\mathcal{H}, \min\{\mathcal{F}\})$
31: **end while**
32: **procedure** PROCESS RETURN OBJECTS
33: Order the final outputs of the minima of f found in the local minimisation step to find the approximate global minimum.
34: **end procedure**
35:
36: **return** the approximate global minimum and a list of all the minima found in the local minimisation step.